81[M, X].—PAUL CONCUS, *Additional Tables for the Evaluation of* $\int_0^\infty x^\beta e^{-x} f(x)\, dx$ *by Gauss-Laguerre Quadrature*, ms. of 8 typewritten pages, deposited in UMT File.

Tables of abscissae and weight coefficients to fifteen places are presented for the Gauss-Laguerre quadrature formula

$$\int_0^\infty x^\beta e^{-x} f(x)\, dx \sim \sum_{k=1}^{N} H_k f(a_k), \quad \text{for} \quad \beta = -\tfrac{2}{3} \text{ and } -\tfrac{1}{3}, \quad \text{and} \quad N = 1(1)15.$$

These tables supplement those presented previously [1] for $\beta = -\frac{1}{4}, -\frac{1}{2}$, and $-\frac{3}{4}$. The same computer program was used, and the accuracy of the tables is the same. The values of $\Gamma(\frac{1}{3})$ and $\Gamma(\frac{2}{3})$ used in the calculations were taken from papers of Zondek [2] and of Sherry and Fulda [3].

AUTHOR'S SUMMARY

1. P. CONCUS, D. CASSATT, G. JAEHNIG & E. MELBY, "Tables for the evaluation of $\int_0^\infty x^\beta e^{-x} f(x)\, dx$ by Gauss-Laguerre quadrature," *Math. Comp.*, v. 17, 1963, p. 245–256.
2. B. ZONDEK, "The values of $\Gamma(\frac{1}{3})$ and $\Gamma(\frac{2}{3})$ and their logarithms accurate to 28 decimals," *MTAC*, v. 9, 1955, p. 24–25.
3. M. E. SHERRY & S. FULDA, "Calculation of gamma functions to high accuracy," *MTAC*, v. 13, 1959, p. 314–315.

82[M, X].—V. I. KRYLOV, *Approximate Calculation of Integrals*, The Macmillan Company, New York, 1962, x + 357 p., 23 cm. Price $12.50.

This is a translation of Krylov's *Priblizhennoe Vychislenie Integralov*, which appeared in 1959. The translation is in the American idiom and is clear and readable.

As the translator remarks, the book might more accurately have been named "Approximate Integration of Functions of One Variable", specifically one *real* variable. Multiple integrals are not treated, except for a short aside in Chapter 7. As a treatise on numerical evaluation of single integrals, this is an excellent book: it is clearly written and comprehensive, developing almost all known practical methods of integration and providing thorough treatments of error estimation and of the question of convergence. A great deal of the material appears for the first time in book form; much had been available before this only in the Russian journals. The text proceeds at a fairly, but not excessively, rapid pace, with frequent pauses to linger over the qualities of a particular formula, or subfamily of formulas, from a wide family under consideration.

The contents are divided into three parts. The first, of about 60 pages, is preparatory, developing certain mathematical topics that will be needed subsequently. These are: The Bernoulli Polynomials, Orthogonal Polynomials, Interpolation, and Banach Spaces. In each case the author moves by a short route from the basic definitions and theorems to the material he will need in his treatment of numerical quadrature; for example, he starts with the definition of Banach space, introduces the most important spaces and linear operators, and proves the uniform boundedness principle, in a very readable chapter only 12 pages long.

The second part, of over 200 pages length, is the heart of the book. It begins, in Chapter 5, with a general discussion of linear quadrature methods and their errors, and derives a general formula for the remainder in approximate quadrature. Chapter 6 takes up interpolatory quadrature methods, and derives the Newton-Cotes formulas and their remainders. The trapezoidal rule, Simpson's rule, and the three-eighths rule are discussed in greater detail. Chapter 7 begins the use of an approach that recurs throughout the book—the construction of formulas of highest degree of precision, sometimes referred to as "Gaussian" formulas. The general theory is set down on the basis of the theory of orthogonal polynomials, and the Gauss-Legendre, Gauss-Jacobi, Gauss-Hermite, and related families, are found, together with their remainder terms. In Chapters 9 and 10 formulas subject to special conditions—the use of some preassigned abscissas, or the restriction to equal coefficients—are treated along similar lines. Chapter 8 is devoted to the so called "best integration formulas"—those defined by regarding the remainder as a linear functional on an appropriate function space, and making its norm minimal. Not much is actually known about these formulas, though they have excited considerable interest. Chapter 11 is devoted mainly to a development of Krylov's: an expansion of the remainder of any quadrature formula in a form similar to the Euler-Maclaurin formula (which is Krylov's formula for the trapezoid rule), and the use of terms of this expansion to improve the accuracy of estimates of integrals. Chapter 12 takes up, in considerable detail, the question of convergence of sequences of quadrature formulas. For analytic integrands a variety of interesting results is proven by function-theoretic methods. Theorems dealing with more general integrands are also given.

The last section of the book, about 55 pages long, is devoted to a topic not usually treated in American texts—numerical indefinite integration; that is, the evaluation of an integral for a considerable number of (equally spaced) values of the upper limit. After an introductory chapter devoted to a very careful discussion of the propagation of errors and stability conditions, two interesting methods due to the author are treated. Each is designed to maximize the degree of precision of the formula while minimizing the number of new values of the integrand to be computed at each step. Some tables of abscissas and coefficients for these methods are given.

The book concludes with appendices listing abscissas and coefficients for the most important Gaussian quadrature formulas. The translator has expanded these tables beyond those given in the original edition.

As this is the only book in English on its subject, it is hardly necessary to recommend it. We are fortunate in the high quality of its writing. The translator has made a real contribution to the English mathematical literature.

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83[P, X, Z].—HOWARD AIKEN & WILLIAM F. MAIN (editors), *Switching Theory in Space Technology*, Stanford University Press, Stanford, California, 1963, x + 357 p., 26 cm. Price $11.50.

This volume consists of twenty-five papers on switching theory, switching circuits, and related topics presented at the Symposium on the Application of Switching Theory in Space Technology, held at Sunnyvale, California, February